King School

Math Review Summer Assignment

For Students Entering MAT 802

2021-2022

Each section has an “In a Nutshell” review followed by practice problems.
-Please read through the review first, then complete the circled problems.
-All work should be done on a separate sheet of paper in order.
-Each problem should be clearly labeled with the page number and problem number.
-Every answer should be circled.
-No final answers without supporting work will be accepted. ANY SUMMER ASSIGNMENT SUBMITED WITHOUT SUFFICIENT ACCOMPANYING WORK WILL NOT BE ACCEPTED AND WILL
REQUIRE YOU TO REDO THE ENTIRE ASSIGNMENT.
Please be sure to use notes from the previous year as well as educational resources on the Internet such as Kahn Academy to refresh your memory if need be.
-Working with a tutor on these exact problems is not permitted.
-This assignment will be handed in the first day of class and will count towards your first graded assignment.
-You will submit all of the pages with your work, but you do not need to submit this packet.

Chapter	Page	Questions
Ch.1	p.23	#3, 5, 11
Ch.2	p.68	#1, 2, 8, 10
Ch.3	p.90	#1-5 All
Ch.4	p.113	#1, 2, 3 (a-b), 4
Ch.5	P.131	#1 (a-f), 2, 3, 4

IN A NUTSHELL

Whole Numbers
0, 1, 2, 3, ...

- **0, 1** (neither prime nor composite)
- **Prime Numbers** **2, 3, 5, 7, ...** (has only two factors, 1 and itself)
- **Composite Numbers** **4, 6, 8, 9, ...** (has more than two factors)

Factors and Multiples

As $24 = 3 \times 8$,
3 and 8 are factors of 24,
and 24 is a multiple of 3 as well as a multiple of 8.

GCF = Greatest Common Factor
LCM = Least Common Multiple

Exponential Notation

8 to the third power → $8^3 = 8 \times 8 \times 8$

exponent

base

The exponent shows how many times the base is used as a factor.

Prime Factorization

$700 = 2^2 \times 5^2 \times 7$
$840 = 2^3 \times 3 \times 5 \times 7$

All bases are prime.

$\text{GCF} = 2^2 \times 5 \times 7$
$\text{LCM} = 2^3 \times 3 \times 5^2 \times 7$

Square and Square Root

$3^2 = 9$ and $\sqrt{9} = 3$

A number whose square root is a whole number is called a perfect square.

Cube and Cube Root

$5^3 = 125$ and $\sqrt[3]{125} = 5$

A number whose cube root is a whole number is called a perfect cube.

REVIEW EXERCISE 1

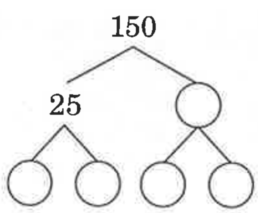
1. Find the smallest number that has 2, 5, and 7 as its prime factors.

2. Find the prime factorization of the greatest 3-digit number.

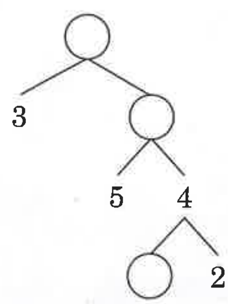
3. Determine whether each number is prime or composite.
   (a) 649 **(b)** 721

4. Determine whether each sentence is true or false.
   (a) If 3 and 5 are factors of a number, then 15 is a factor of the number.
   (b) If 246 is a multiple of a number, then 123 is a multiple of the number.

5. **(a)** Complete the following factor trees.
   (i)



   (ii)



   (b) Write down the prime factorization of the number at the top of each tree.
   (c) Find the GCF and LCM of the numbers at the top of the trees.

6. **(a)** Find the GCF of 12, 40, and 45.
   (b) Find the LCM of 12, 40, and 45.
   (c) Find the greatest 4-digit number which is a common multiple of 12, 40, and 45.

7. **(a)** Find the prime factorization of
   (i) 12,
   (ii) 144,
   (iii) 5,040.
   (b) The GCF and LCM of two numbers are 12 and 5,040 respectively. If one of the numbers is 144, find the other number.

8. The dimensions of a rectangle are $(2^5 \times 7)$ cm by $(2 \times 5^2 \times 7^3)$ cm.
   (a) Find the area of the rectangle, giving your answer in prime factorization form.
   (b) A square has the same area as the rectangle. Find the length of a side of the square.

9. Using prime factorization, find
   (a) the value of $\sqrt{1{,}521}$,
   (b) the value of $\sqrt[3]{375 \times 243}$,
   (c) the GCF of the results in **(a)** and **(b)**.

10. A bell rings every 25 minutes while another bell rings every 40 minutes. If the bells rang together at 6 A.M., at what time would they next ring together?

11. A box contains an assortment of three types of chocolate bars. It has 18 bars with almonds, 24 bars with hazelnuts, and 30 bars with peanuts. The chocolate bars are shared among some students. Each student gets only one type of chocolate bar and every student gets the same number of chocolate bars. If each student gets the greatest number of chocolate bars,
    (a) how many chocolate bars does each student get?
    (b) how many students will get chocolate bars with peanuts?

12. A rectangular board measures 630 cm by 396 cm. It is divided into small squares of equal size.
   (a) Find
      (i) the largest possible length of the side of a square,
      (ii) the least total number of squares.
   (b) (i) What is the next largest length of the side of a squ
      (ii) Find the total number of in this case.

EXTEND YOUR LEARNING CURVE

Sieve of Eratosthenes

(a) Use the Sieve of Eratosthenes to find all the prime numbers less than 200.

(b) Compare the numbers of primes in the intervals 0–99 and 100–199.

(c) Would the number of primes in the interval 200–299 be less than that in 100–199? Explain briefly.

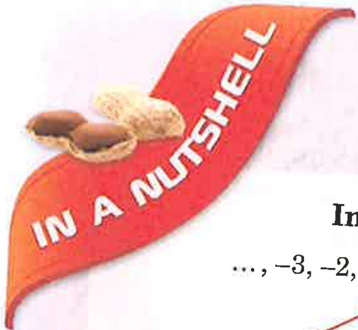
WRITE IN YOUR JOURNAL

1. Share the difficulties you have encountered when you use prime factorization to find the GCF and LCM of a set of numbers.

2. Give one example each of the relevance of LCM and GCF in real-life situations.

17. Michael's time for the 100-meter sprint is recorded as 9.96 seconds. State two possible values of his actual time for the 100-meter sprint.

18. Find some examples of the use of deci in the media. Are the numbers exac estimated? If they are estimated, what their degrees of accuracy?



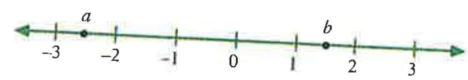
Integers

$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Negative integers	Zero	Positive integers
$\ldots, -3, -2, -1$	0	$1, 2, 3, \ldots$

Number Line

- A number line shows the order of real numbers.



- a is to the left of b.
  $\therefore\ a < b$

Addition and Subtraction of Integers

$$a + b = a + b$$
$$-a + (-b) = -(a + b)$$
$$-a + b = -(a - b) \quad \text{if } a \geqslant b$$
$$-a + b = b - a \quad \text{if } b > a$$
$$a + (-b) = a - b \quad \text{if } a \geqslant b$$
$$a + (-b) = -(b - a) \quad \text{if } b > a$$

To subtract, we add the additive inverse.

Order of Operations

E.g., $-7 + 3^2 \times (5 - 7)$
$= -7 + 3^2 \times (-2)$ parentheses
$= -7 + 9 \times (-2)$ powers
$= -7 + (-18)$ × and ÷
$= -25$ + and –

Additive Inverse

The additive inverse, or opposite, of a number is the number when added to it yields 0.
E.g., the additive inverse of -5 is 5.

Multiplicative Inverse

The multiplicative inverse, or reciprocal, of a number is that number when multiplied by it yields 1.

E.g., the multiplicative inverse of -5 is $-\frac{1}{5}$.

IN A NUTSHELL

Absolute Value

The absolute value of a number is the distance of the number from 0 on the number line.

$|x| = x$ if $x \geqslant 0$, e.g., $|7| = 7$, $|0| = 0$

$|x| = -x$ if $x < 0$, e.g., $|-4| = 4$

Multiplication and Division of Integers

If two numbers have the same sign, their product and their quotient are positive.

If two numbers have opposite signs, their product and their quotient are negative.

Rational Numbers

They can be expressed in the form $\frac{a}{b}$, where a, b are integers, and $b \neq 0$.

Rational Numbers

- **Integers**
  e.g., 3, –4, 0
- **Fractions**
  e.g., $\frac{2}{3}$, $-\frac{5}{6}$

Real Numbers

They are all the numbers on the number line.

Real Numbers

- **Rational Numbers**
  - **Terminating decimals**
    e.g., 0.25, 0.36
  - **Repeating decimals**
    e.g., $0.\overline{3}$, $0.\overline{78}$
- **Irrational Numbers**
  - **Non-terminating and non-repeating decimals**
    e.g., $\sqrt{2}$, $\sqrt{3}$

Approximation by Rounding

Rule: If the digit under consideration is 4 or less, you round the number down. If it is 5 or more, round it up.

Rounding to whole numbers

For example, 34,267 = 34,000 (correct to the nearest 1,000)

Rounding to decimal places

For example, 6.0784 = 6.08 (correct to 2 d.p.)

REVIEW EXERCISE 2

1. Complete the following addition table.

+	8	1	
2			
	3	−4	−7
−3			−5

2. Complete the following multiplication table.

×	−3		
	15		
−2		8	
7			−42

3. **(a)** The night temperature in the Gobi Desert was −22°F after it has dropped 14°F. What was the initial temperature before the decrease?

   (b) On another day, the temperature in the desert at 12 midnight was 14.6°F. The temperature decreased constantly during that day at 2°F every hour. What was the temperature in the desert at 6 A.M. on that day?

4.

Location	Altitude
Death Valley, The United States	86 m below sea level
Turfan Depression, China	154 m below sea level
Mount Everest, Nepal	8,848 m above sea level

Using the information in the table above, find the difference in altitude between

(a) Death Valley and Turfan Depression,

(b) Turfan Depression and Mount Everest.

5. **(a)** Find the value of $(-2)^2$ and $\left(\frac{7}{6}\right)^2$.

   (b) Represent the numbers -2, $\frac{7}{6}$, $(-2)^2$, and $\left(\frac{7}{6}\right)^2$ on a number line.

   (c) Fill in ☐ with "<", ">", or "=" to make a true statement.

   (i) -2 ☐ $\frac{7}{6}$

   (ii) $(-2)^2$ ☐ $\left(\frac{7}{6}\right)^2$

6. A quality control supervisor measures the actual volumes of six packets of fruit juice. Each packet of fruit juice is supposed to contain 375 mL of juice. The table shows the inspection results.

Packet	1	2	3	4	5	6
Amount below or above the required volume (mL)	−5	+12	−6	−9	+7	−2

   (a) Find the actual volumes of juice in packet 1 and packet 2.

   (b) Find the total volume of juice in the six packets.

7. There are four participants in a golf tournament. Their scores (which are the number of strokes below or above a standard value) in three rounds are shown in the following table.

	Alex	Ben	Charles	Dave
Round 1	−5	+3	+10	−3
Round 2	−1	−2	+2	−4
Round 3	−2	−1	−3	+5

   (a) Who had the lowest score in Round 1?

   (b) Find the total score of each person at the end of three rounds.

   (c) The winner in the tournament was the person with the lowest total score. Who was the winner?

8. Evaluate the following.
   (a) $(-16) \times (-3) - (-8) \times 5$
   (b) $[-2 + (-7)]^3$
   (c) $3\frac{1}{2} \times \left(-5\frac{1}{7}\right) \div 1\frac{4}{5}$
   (d) $\frac{2}{3} \times \left(\frac{1}{4} - \frac{1}{8}\right)^2$

9. Evaluate the following.
   (a) $\left|4 \times \left(-2\frac{3}{8}\right)\right|$
   (b) $-\left|(-2)^2\right|$
   (c) $\frac{-3}{|-15|} \times \left|(-5)^3\right|$

10. New York City, the most populous city in the United States, has a population of 8,263,710 and a land area of 304.71 square miles. What is the average number of people per square mile of the city? Give your answer correct to
   (a) the nearest integer,
   (b) the nearest hundred.

11. In year 2011, the population of Singapore was 5,183,700 and the total number of mobile phone subscribers was 7,443,800. Find the average number of mobile phones subscribed per person in Singapore, giving your answer correct to
   (a) 2 decimal places,
   (b) 1 decimal place.

12. The world is divided into 24 one-hour time zones. The table below shows the time difference between the local times of London and some cities. A difference of "+10" means the time is 10 hours ahead of the London time while "–10" means 10 hours behind the London time.

City	Time difference (hours) from London
Auckland	+13
Los Angeles	–8
New York	–5
Singapore	+8

   (a) Which two cities in the table have the greatest time difference? What is the difference?
   (b) What is the time difference between each of the following pairs of cities?
      (i) Los Angeles and Singapore,
      (ii) New York and Los Angeles.
   (c) For each pair of cities in part **(b)**, state if the time of the first city is ahead of or behind that of the second city.
   (d) When Auckland ushered in New Year at 12.00 A.M. on January 1, 2012, what was the date and local time in New York?

IN A NUTSHELL

Basic Notations

In algebra, symbols are used to represent numbers.

$$\text{sum} = a + b$$
$$\text{difference} = a - b$$
$$\text{product} = ab \text{ or } a \times b$$
$$\text{quotient} = \frac{a}{b} \text{ or } a \div b$$

(For division, $b \neq 0$.)

$$a \times 1 = 1 \times a = a$$
$$a \times 3 = 3a = a + a + a$$
$$a \times a = a^2$$
$$a \times a \times a = a^3$$

Algebraic Expressions

E.g., $2n + 1$
$3a + 4b - 5c$

Substitution

When $n = 3$,

$$2n + 1 = 2(3) + 1$$
$$= 7$$

Formulas

E.g., $A = \pi r^2$
$P = 2(a + b)$

Substitution

When $a = 3$ and $b = 4$,

$$P = 2(3 + 4)$$
$$= 14$$

Representing Real-world Situations using Algebraic Expressions

Let x be a variable for a quantity in the situation. Then, express other quantities in the situation in terms of x.

REVIEW EXERCISE 3

1. Simplify the following.
   (a) $5s \times 3t + 1 \times u$
   (b) $m - 4n \times 6m \times m$
   (c) $(a \times 4 - b \times b) \div 2c$
   (d) $3x - b \div c - 5 \times y$

2. Express the following word statements algebraically.
   (a) Subtract $c \times c$ from $d \times 5$.
   (b) Divide x cubed by y squared.
   (c) Divide the product of $6a$ and 4 by $8b$.

3. Given the formula $E = \frac{1}{2}m(v^2 - u^2)$, find the value of E when $m = 5$, $v = 11$ and $u = 7$.

4. Given the formula $y = \frac{a - 3b^2}{(a - 3b)^2}$, find the value of y when $a = 10$ and $b = 2$.

5. The capacity of a car is 5 passengers and that of a van is 8 passengers. Find the total capacity for m cars and n vans.

6. Find the value of the expression $(2a + 3b)^2$ when $a = 1$ and $b = -2$.

7. A piece of wire is 100 centimeters long. One piece of a centimeters and two pieces of b centimeters each are cut from it.
   (a) Express the length of the remaining part in terms of a and b.
   (b) (i) If the remaining part is bent into a square, express the length of one side of the square in terms of a and b.
   (ii) When $a = 24$ and $b = 16$, find the length of one side of the square.

8. There are x boys and y girls in a class. Half of the boys and $\frac{1}{3}$ of the girls join a math club.
   (a) Express, in terms of x and y,
   (i) the total number of students in the class,
   (ii) the total number of students joining the math club.
   (b) When $x = 18$ and $y = 24$, find the number of students joining the math club.

9. John's savings after n months is
   $$\$(2{,}500 + 300n).$$
   (a) Find the amount of savings he has after
   (i) 5 months,
   (ii) 1 year.
   (b) After n months, John uses all his savings to buy gold coins costing $\$g$ each.
   (i) Express the number of gold coins he buys in terms of g and n.
   (ii) When $g = 100$ and $n = 6$, find the number of gold coins John buys.

10. Helen is h years old now.
    (a) Find, in terms of h, Helen's age
    (i) 3 years ago,
    (ii) in 3 years' time.
    Helen's mother is now 4 times as old as Helen was 3 years ago. Her sister's present age is 2 years more than one-third of Helen's age in 3 years' time.
    (b) How old is Helen's mother now in terms of h?
    (c) What is the present age of Helen's sister in terms of h?
    (d) If Helen is 12 years old, how old are her mother and her sister?

11. Mrs. Jones went shopping, and bought a dress, a skirt, and a diamond ring. The price of the dress is $80 more than the price of the skirt, and the price of the ring is 25 times as much as that of the dress. Let $$k$ be the price of the skirt.
    (a) Express the price of the dress in terms of k.
    (b) Express the price of the ring in terms of k.
    (c) Given that the total sum of money Mrs. Jones spent in this shopping spree is $$T$, write a formula connecting T and k.
    (d) If the skirt costs $49, find the total sum Mrs. Jones spent.

12. Yvonne saves her nickels, dimes, and quarters in a box. The number of dimes in the box is 30 more than the number of nickels, and the number of quarters is 15 less than twice the number of dimes. Let x, y, and z be the number of nickels, dimes, and quarters Yvonne saves respectively.
    (a) Express y and z each in terms of x.
    (b) How many dimes and quarters are there in the box if there are 18 nickels?
    (c) What is the total sum of money Yvonne saves?

EXTEND YOUR LEARNING CURVE

Hand-shaking Problem

There are n persons attending a party. Each person shakes hands with every other person just once. Let T be the total number of handshakes.

(a) Complete the following table.

n	2	3	4	5	6
T					

(b) Establish a formula connecting n and T.

WRITE IN YOUR JOURNAL

Julie says that $3a^2 + 2a + 5a^2 = 10a^3$

and $7b \times 5b^2 \times 6 = 210b$.

Do you agree with Julie? If not, what are the mistakes in her algebraic manipulations?

IN A NUTSHELL

Terms

There are 3 terms in the following expression.

$$\underbrace{3a}_{\text{term}} \quad \underbrace{-2bc}_{\text{term}} \quad \underbrace{+5}_{\text{term}}$$

constant term

coefficient of bc is -2

coefficient of a is 3

Like Terms and Unlike Terms

Like Terms	Unlike Terms
$3x, -4x$	$3x, 4y$
$6xy, 7xy$	$6ab, 6ac$

- An expression can be simplified by collecting like terms.

  For example,
  $3x - 2y - 5x + 6y$
  $= 3x - 5x - 2y + 6y$
  $= -2x + 4y$

Distributive Law

$$a(x + y) = ax + ay$$
$$a(x - y) = ax - ay$$

For example,

$$4(-2x + 3y) = 4(-2x) + 4(3y)$$
$$= -8x + 12y$$
$$-(x + y) = -x - y$$
$$-(x - y) = -x + y$$

Addition and Subtraction of Linear Expressions

Addition and subtraction are carried out by removing parentheses and collecting like terms.

For example,
$2(-2x + 3y) - (-3x + 4y)$
$= -4x + 6y + 3x - 4y$
$= -4x + 3x + 6y - 4y$
$= -x + 2y$

Factorization

- Write an algebraic expression as a product of its factors.
- Factoring out common factor(s)

  $$ax + ay = a(x + y)$$

  For example, $12ax - 15ay + 3a$
  $= 3a(4x) - 3a(5y) + 3a(1)$
  $= 3a(4x - 5y + 1)$

- Factoring by grouping
  $ax + ay + bx + by = a(x + y) + b(x + y)$
  $= (a + b)(x + y)$

  For example, $3mx + 6my - nx - 2ny$
  $= (3mx + 6my) - (nx + 2ny)$
  $= 3m(x + 2y) - n(x + 2y)$
  $= (3m - n)(x + 2y)$

Order of Operation

The rules are :

1. Simplify all operations inside Parentheses.
2. Simplify all Exponents.
3. Multiply and Divide, working from left to right.
4. Add and Subtract, working from left to right.

e.g., $-7 + 3^2 \times (5 - 7)$
$= -7 + 9 \times (-2)$
$= -7 + (-18)$
$= -7 - 18$
$= -25$

REVIEW EXERCISE 4

1. Simplify the following.
   (a) $2a + 3b - 4a + 5b$
   (b) $(2p - 7q - 6r) + (3p - 4q - r)$
   (c) $(2x - 3y + 4) + (-3x + 6y - 1)$
   (d) $(-3m - 8n + 2p) - (-4m + 7n - 3p)$

2. Simplify the following.
   (a) $4(2m - 1) + 3(4m + 1)$
   (b) $3(2p - 7q) - 6(p + 3q)$
   (c) $-3(4x + y) + 2(8x - 5y)$
   (d) $-7(2x - y + 3) + -4(-3x + y - 5)$

3. Express each of the following as a single fraction in its simplest form.
   (a) $\dfrac{3x}{4} + \dfrac{2(1-3x)}{5}$
   (b) $\dfrac{4(2x-1)}{7} - \dfrac{x-3}{2}$
   (c) $-\dfrac{2(5-x)}{3} - \dfrac{7(2x+1)}{9}$
   (d) $1 - \dfrac{(x+2)}{6} + \dfrac{4(1-2x)}{5}$

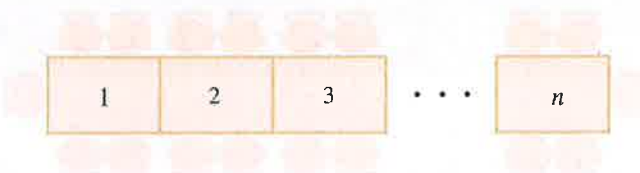
4. Factor the following completely.
   (a) $5ac - c$
   (b) $12xy + 36x$
   (c) $-9pq - 15pr$
   (d) $15ax - 20ay + 10az$

5. **(a)** Simplify $6(x + 2y) - 7(4x - 3y)$.
   (b) Factor the result in **(a)**.
   (c) When $x = -1$ and $y = 5$, find the value of the given expression.

6. The numbers of marbles in two bags are $a(3x - y)$ and $2b(3x - y)$.
   (a) Find the total number of marbles in the bags.
   (b) Factor the result in **(a)**.
   (c) All the marbles are arranged in rows and columns to form a rectangle. If one side of the rectangle has $(a + 2b)$ marbles, find the number of marbles on the other side.

7. A test consists of three parts. The minimum total score required to pass the test is $(8x + 4y)$ points. Jacob scores $(2x - y + 10)$ points and $(2x + 3y - 6)$ points in the first two parts.
   (a) Find Jacob's total score in the first two parts.
   (b) How many points does Jacob score in the third parts if he just passes the test?
   (c) Factor the result in **(b)**.

8. In the diagram, n identical tables are joined end-to-end to form a long table. A single table can have 2 seats on each side and one seat at each end.



   (a) Copy and complete the following table.

n	1	2	3	4	5
Total number of seats					

   (b) If n tables are joined to form a long table, express the total number of seats in terms of n.

MATH@WORK

13. Mrs. Perry has a sum of money to buy fruits. She can buy n mangoes at \$1.60 each and have \$0.80 left. Alternatively, she can buy $(n + 10)$ apples at \$0.70 each and have \$0.10 left.

(a) Find the value of n.

(b) How much money does Mrs. Perry have for buying fruits?

(c) If Mrs. Perry buys 3 mangoes and uses the rest of the money to buy apples,

(i) how many apples can she buy?

(ii) how much money will she have left?

14. George wants to participate in a triathlon competition where he has to swim, cycle and run some distances. The cycling distance is 4 times the running distance. The swimming distance is 5.5 miles less than the running distance, and is 24 miles less than the cycling distance. Find the total distance of the race.

BRAIN WORKS

15. Write an application problem such that the equation to be formed for solving the problem is $5x + 4(x - 10) = 140$.

IN A NUTSHELL

Equation

An equation is an equality that involves one or more variables.

Equations involving parentheses

Remove the parentheses first when solving such equations.

E.g.,
$$\begin{aligned} 3(4x + 1) &= 7 + 2(5x - 6) \\ 12x + 3 &= 7 + 10x - 12 \\ 12x - 10x &= -5 - 3 \\ 2x &= -8 \\ x &= -4 \end{aligned}$$

Problem solving involving linear equations

You can use the general strategy to solve word problems as follows:

- Read the question carefully and identify the unknown quantity.
- Use a letter x to represent the unknown quantity.
- Express other quantities in terms of x.
- Form an equation based on the given information.
- Solve the equation.
- Write down the answer statement.

Linear equation in one variable x

E.g., $ax + b = 0$ where a, b are constants and $a \neq 0$.

Methods of solving linear equations in one variable

(a) Add a term to both sides

E.g.,
$$\begin{aligned} x - 3 &= 6 \\ x - 3 + 3 &= 6 + 3 \\ x &= 9 \end{aligned}$$

(b) Subtract a term from both sides

E.g.,
$$\begin{aligned} x + 2 &= 14 \\ x + 2 - 2 &= 14 - 2 \\ x &= 12 \end{aligned}$$

(c) Multiply both sides by a constant

E.g.,
$$\begin{aligned} \frac{x}{5} &= 6 \\ \frac{x}{5} \times 5 &= 6 \times 5 \\ x &= 30 \end{aligned}$$

(d) Divide both sides by a constant

E.g.,
$$\begin{aligned} 4x &= 12 \\ \frac{4x}{4} &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

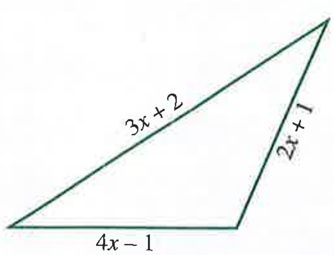
REVIEW EXERCISE 5

1. Solve the following equations.
   (a) $13x - 22 = 30$
   (b) $2(5x - 8) + 6 = 11$
   (c) $\frac{2x}{3} + \frac{x}{5} = 13$
   (d) $1 - \frac{4}{7}x = 23 + x$
   (e) $\frac{4x-5}{2} = \frac{7x-3}{9}$
   (f) $\frac{x-4}{3} - \frac{2x+1}{6} = \frac{5x-1}{2}$
   (g) $\frac{2}{x-7} = 6$
   (h) $\frac{4x-1}{5x+1} = \frac{5}{7}$

2. Given the formula $D = b^2 - 4ac$,
   (a) find the value of D when $a = 1$, $b = -5$, and $c = 3$,
   (b) find the value of c when $a = 2$, $b = 3$, and $D = 49$.

3. Given the formula $S = \frac{n(a+b)}{2}$,
   (a) find the value of S when $a = 1$, $b = 25$, and $n = 12$,
   (b) find the value of a when $b = 41$, $n = 15$, and $S = 330$.

4. The lengths of the sides of a triangle are $(2x + 1)$ cm, $(3x + 2)$ cm, and $(4x - 1)$ cm.
   (a) Find the perimeter of the triangle in terms of x.
   (b) If the perimeter of the triangle is 47 cm, find the value of x.



5. Peter has 96 stamps and Sam has 63. How many stamps should Sam give Peter so that Peter will have twice as many stamps as Sam?

6. A boy is 26 years younger than his father. In 3 years' time, his age will be $\frac{1}{3}$ his father's age. Find the boy's present age.

7. The price of a skirt is \$25 more than the price of a T-shirt. The total price of 3 skirts and 8 T-shirts is \$339. Find the price of a skirt.

8. In a certain week, the amount of time Lisa spent on watching television was 3 hours more than twice the time she spent on doing her mathematics homework. If the total time she spent on these two activities was 30 hours in that week, how many hours did Lisa spend on doing her mathematics homework?

9. The number of books in a class library is 17 more than 3 times the number of students in the class. If 5 students are absent, each student can borrow exactly 4 books from the library. Find the number of students in the class.

10. A number is 4 times greater than another number. By subtracting 3 from each number, the first number becomes 5 times greater than the second. What are the two numbers?

IN A NUTSHELL

Ratio

- A ratio is a comparison of two similar quantities.

$$a : b = \frac{a}{b} \quad (b \neq 0)$$

- It has no unit.
- $$a : b = ma : mb,$$
  $$a : b = \frac{a}{m} : \frac{b}{m} \ (m \neq 0),$$
  $$a : b : c = ma : mb : mc, \text{ and}$$
  $$a : b : c = \frac{a}{m} : \frac{b}{m} : \frac{c}{m} \ (m \neq 0),$$
  are equivalent ratios.

Rate

- It is a comparison of two quantities by division. It is usually expressed as one quantity per unit of another quantity.

  E.g., $\frac{\$10}{4 \text{ kg}} = \$2.50/\text{kg}$

Speed

- Average speed = $\frac{\text{Total distance traveled}}{\text{Total time taken}}$
- Conversion of units

$$10 \text{ m/s} \xrightarrow{\times \frac{3{,}600}{1{,}000}} 36 \text{ km/h}$$

$$10 \text{ m/s} = 36 \text{ km/h}$$

$$36 \text{ km/h} \xrightarrow{\times \frac{1{,}000}{3{,}600}} 10 \text{ m/s}$$

1 mph is about 1.61 km/hr.

REVIEW EXERCISE 6

1. Two cubes are of sides 6 inches and 8 inches respectively. Find the ratio of
   (a) their sides,
   (b) their areas on one face,
   (c) their volumes.

2. Car X travels 60 miles in 45 minutes. Car Y travels 72 miles in 1 hour and 20 minutes. Find
   (a) the average speed of Car X in mph.
   (b) the average speed of Car Y in mph.
   (c) the ratio of the average speed of Car X to that of Car Y.

3. There are a total of 240 pieces of \$5 bills and \$10 bills. The numbers of \$5 and \$10 bills are in the ratio 3 : 2. Find
   (a) the number of \$5 bills,
   (b) the number of \$10 bills,
   (c) the ratio of the value of the \$5 bills to that of the \$10 bills.

4. Syrup and water are mixed in the ratio 1 : 4 by volume. If the volume of the solution is 600 gallons,
   (a) find the volume of syrup in the solution,
   (b) find the volume of water in the solution,
   (c) how much syrup must be added to the solution so that the ratio of volume of syrup to volume of water in the solution becomes 1 : 3?

5. A metal bar of mass 3.6 kilograms is cut into two pieces in the ratio 3 : 5. The length of the shorter piece is 45 centimeters. Find
   (a) the length of the longer piece,
   (b) the length of the original metal bar,
   (c) the mass per unit length of the bar in kg/m,
   (d) the mass of the shorter piece.

6. **(a)** Simplify each of the following ratios.
   (i) $a : b = 1\frac{1}{2} : 2\frac{2}{5}$
   (ii) $b : c = 0.105 : 0.350$
   (b) Find the ratio $a : b : c$.
   (c) Alan, Bob and Cathy share \$500 in the ratio $a : b : c$ found in **(b)**. Find Alan's share, correct to 2 decimal places.

7. The prices of two stocks, A and B, are in the ratio 2 : 3. If the price of A increases by \$12 while the price of B decreases by \$6, the ratio of their prices becomes 10 : 11. Find the original prices of the stocks.

8. A 2-liter bottle of canola oil is sold for \$15. A $2\frac{1}{2}$-liter bottle of olive oil is sold for \$30.
   (a) Find the unit price x of canola oil in \$/l.
   (b) Find the unit price y of olive oil in \$/l.
   (c) Find the ratio $x : y$.
   (d) Suppose both types of oil are equally good for cooking, which one is a better buy?
   (e) The bottle of canola oil can be used for 16 days. Find its consumption rate in l/day.
   (f) If the consumption rate of the bottle of olive oil is the same as that of canola oil, how many days can it last?

9. A man took $2\frac{1}{2}$ hours to drive 195 km from San Diego to Los Angeles. He used 20 liters of gasoline for the entire journey.
   (a) Find his average speed.
   (b) Find the gasoline consumption rate in km/l.
   (c) If he drove at an average speed of 110 km/h on a highway for 45 minutes during his journey, find his average speed for the remaining part of his journey.

10. A car starts from rest. After traveling 125 meters in 10 seconds, its speed picks up to 25 m/s. It travels at this speed for 20 seconds. Then brakes are applied. The car stops in 6 seconds and the braking distance is 95 meters.
   (a) Express the speed 25 m/s in km/h.
   (b) Find the average speed of the car during the period at which its speed increases.
   (c) Find the average speed of the car during the period the brakes were applied.
   (d) Find the average speed of the car for the whole journey.

11. Towns P and Q are 120 miles apart. Mr. Miller drove from P to Q and was scheduled to reach Q after 2 hours. His average speed was 54 mph for the first 40 minutes.
   (a) What was his average speed for the remaining journey if he managed to arrive just on time?
   (b) The time taken for his return journey is 2 hours and 10 minutes. Find his average speed for
      (i) the return journey,
      (ii) the whole trip.

EXTEND YOUR LEARNING CURVE

Fastest Train

Find out the speed of the fastest train in the world in both km/hr and mph.

WRITE IN YOUR JOURNAL

1. In your own words, describe what a ratio is.
2. What is the difference between rate and speed?

IN A NUTSHELL

Meaning of Percentage

$$n\% = \frac{n}{100}$$

$$1\% = \frac{1}{100}$$

$$100\% = 1$$

Expressing One Quantity as a Percentage of Another and Reverse Percentage

If a is $n\%$ of b,

then $a = \frac{n}{100} \times b$

and $b = \frac{100a}{n}$.

Percentage Increase

Increase = Increased value – Original value

$$\text{Percentage increase} = \frac{\text{Increase}}{\text{Original value}} \times 100\%$$

Increased value = (100% + Increase %) × Original value

Percentage Decrease

Decrease = Original value – Decreased value

$$\text{Percentage decrease} = \frac{\text{Decrease}}{\text{Original value}} \times 100\%$$

Decreased value = (100% – Decrease %) × Original value

Discount

Discount = Marked price – Selling price

$$\text{Percentage discount} = \frac{\text{Discount}}{\text{Marked price}} \times 100\%$$

Selling price = (100% – Discount %) × Marked price

Tax

Tax = Tax rate × Cost

REVIEW EXERCISE 7

1. In a particular month, Mr. Clark spent \$900 on food which was 30% of his monthly salary.
   (a) Find his monthly salary.
   (b) If he saved 12% of his monthly salary, find his monthly savings.

2. A piece of alloy contains 60% copper and 40% zinc by weight. The weight of the alloy is 25 kg.
   (a) Find the weight of
   (i) copper,
   (ii) zinc,
   in the alloy.
   (b) If 5 kg of tin is added to the alloy, find the ratio of the weights of copper, zinc and tin in the new alloy.

3. In a box of pens, 26% of them are red, 38% are blue and the remaining ones are black. There are 57 blue pens in the box.
   (a) Find the total number of pens in the box.
   (b) How many of them are black?
   (c) If 10 more blue pens were put into the box, what would the new percentage of blue pens be?

4. 30% of the members of a club are female and 20% of them own cars. 30 female members in the club own cars.
   (a) How many female members are there?
   (b) What is the total number of members?
   (c) How many new female members have to be recruited to increase the percentage of female members to 37.5%?

5. Richard earned \$36,000 in the year 2010 and \$38,880 in the year 2011.
   (a) Find the percentage increase in his annual income in the year 2011.
   (b) If Richard's income in the year 2010 was 10% less than that of the previous year, what was his annual income in the year 2009?

6. Mr. Wood's monthly expenses was \$2,125 after he had reduced it by 15%. The original monthly expenses was 80% of his monthly income.
   (a) Find the original monthly expenses.
   (b) Find his monthly income.

7. Mrs. Delano's weight increased by 20% to 132 lb.
   (a) Find her original weight.
   (b) If her weight then dropped by 15%, find her new weight.
   (c) What percentage is her new weight of her original weight?

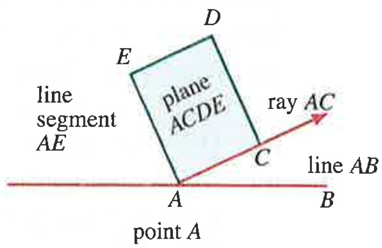
8. The costs of labor and material used in making a table are in the ratio 2 : 3. The cost of the table, which is \$600, is the sum of these two costs.
   (a) Find the cost of
   (i) labor,
   (ii) material used.
   (b) If the cost of labor is increased by 15% while the cost of material is decreased by 20%, find the percentage change in the cost of making the table.

9. A newsagent overstocks 10 copies of a magazine. Eight of these copies are sold at 30% discount and the selling price is \$5.60 each. The remaining two copies are sold at 40% discount.
   (a) Find the marked price of the magazine.
   (b) Find the selling price of each of the last two copies.
   (c) What is the overall percentage discount for the 10 copies of these magazines?

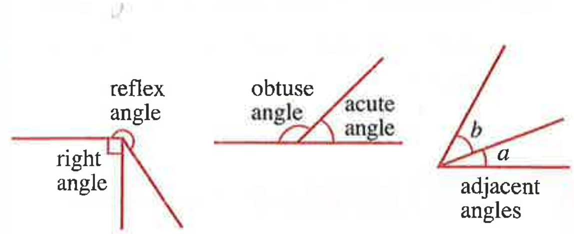
10. In 500 cm^3 of alcohol solution, the percentage of alcohol by volume is 80%.
   (a) Find the volume of alcohol in the solution.
   (b) Find the volume of water that must be added to the solution so that the ratio of the volume of alcohol to that of water becomes 5 : 2.

IN A NUTSHELL

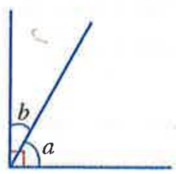
Points, Lines, and Planes



Types of Angles

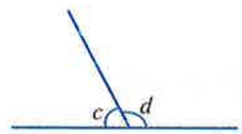


Complementary Angles



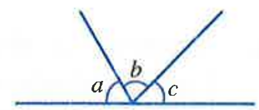
$m\angle a + m\angle b = 90°$

Supplementary Angles



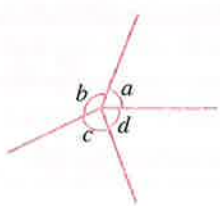
$m\angle c + m\angle d = 180°$

Adjacent Angles on a Straight Line



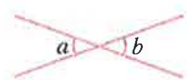
$m\angle a + m\angle b + m\angle c = 180°$
(adj. $\angle$s on a st. line)

Angles at a Point



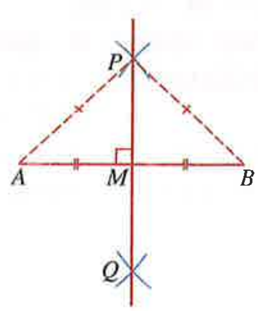
$m\angle a + m\angle b + m\angle c + m\angle d = 360°$
($\angle$s at a point)

Vertically Opposite Angles



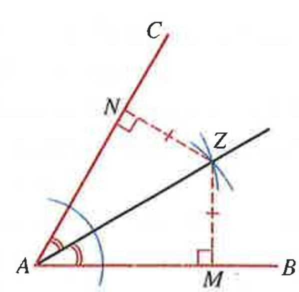
$m\angle a = m\angle b$ (vert. opp. $\angle$s)

Perpendicular Bisector



$AP = BP$ ($\perp$ bisector)
$AM = BM$
$m\angle AMP = m\angle BMP = 90°$

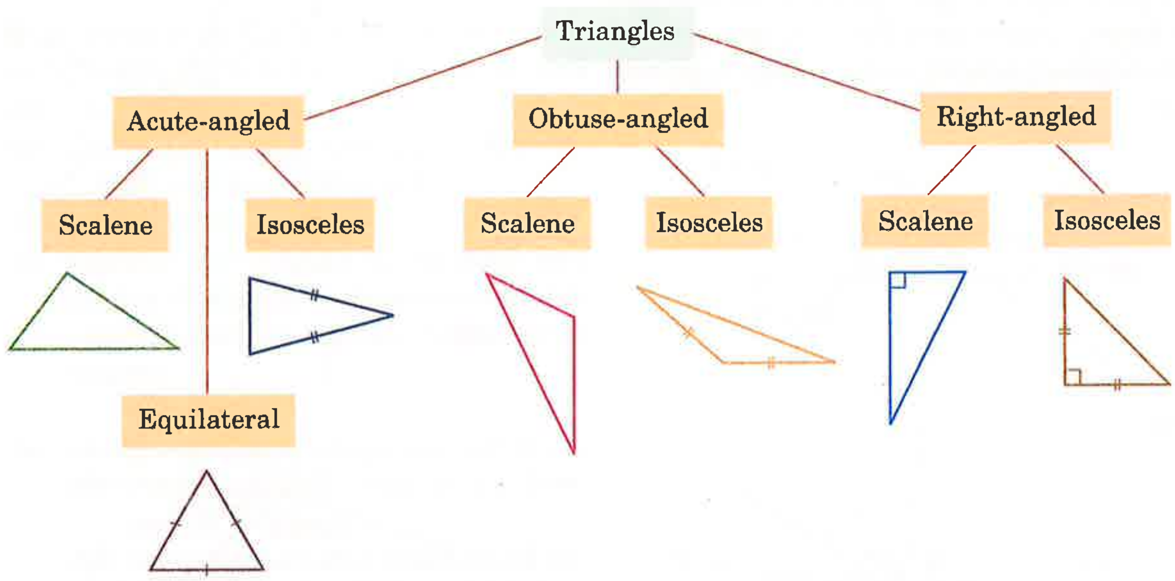
Angle Bisector



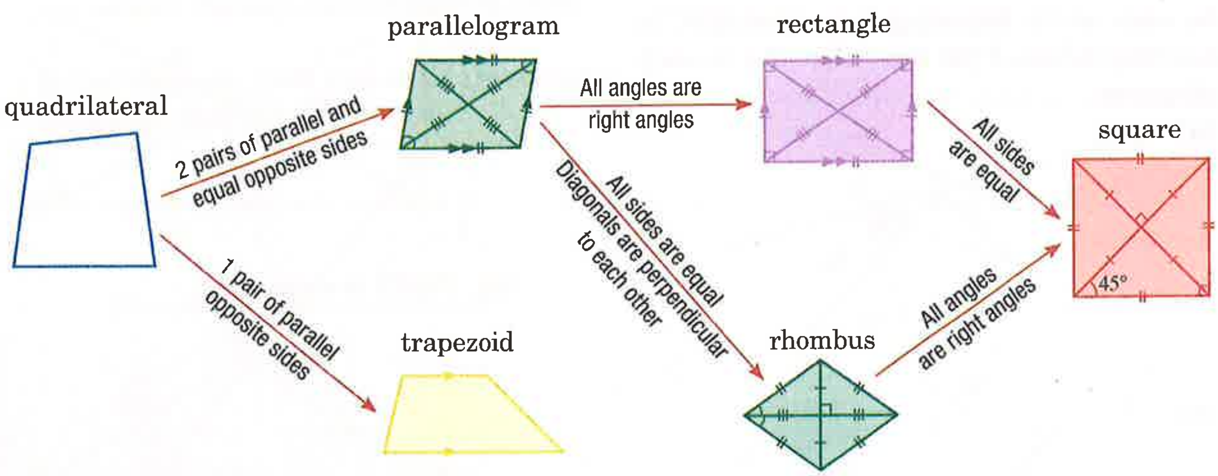
$MZ = NZ$ ($\angle$ bisector)
$m\angle BAZ = m\angle CAZ$

Triangles

Triangles can be classified according to the types of angles and the number of their equal sides.



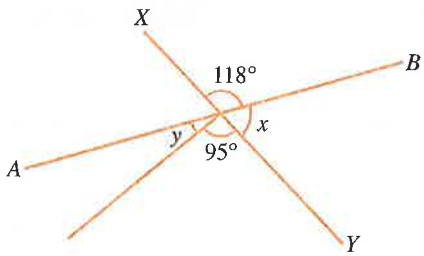
Quadrilaterals



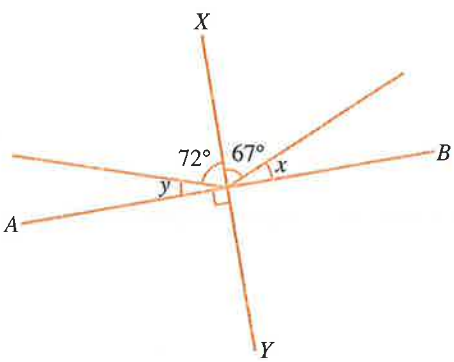
REVIEW EXERCISE 8

1. (a) Given that the angles $5p°$ and $(3p - 20)°$ are supplementary, find the value of p.
   (b) Given that the angles $(33 - q)°$ and $(3q + 5)°$ are complementary, find the value of q.

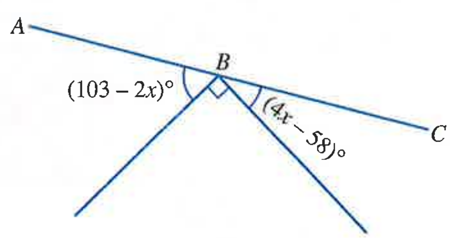
2. In each of the following diagrams, AB and XY are straight lines. Find the measures of the angles marked x and y in each diagram.
   (a)



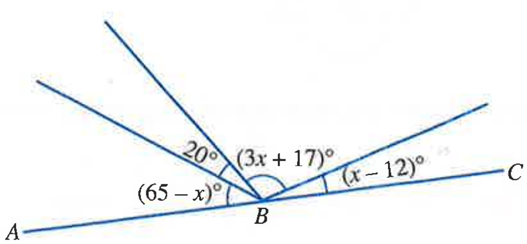
   (b)



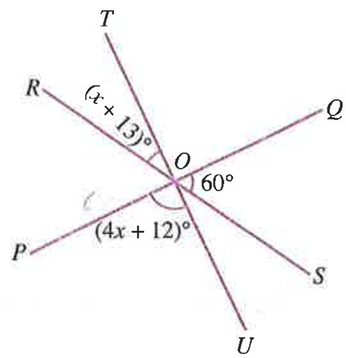
3. In each of the following diagrams, ABC is a straight line. Find the value of x in each diagram.
   (a)



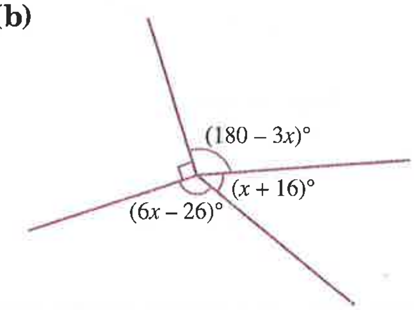
   (b)



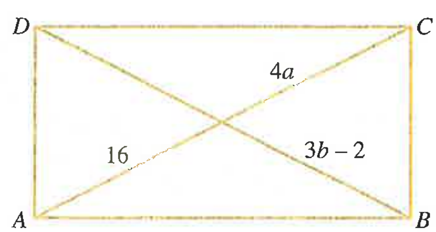
4. Find the value of x in each of the following diagram.
   (a) Straight lines PQ, RS, and TU intersect at O.



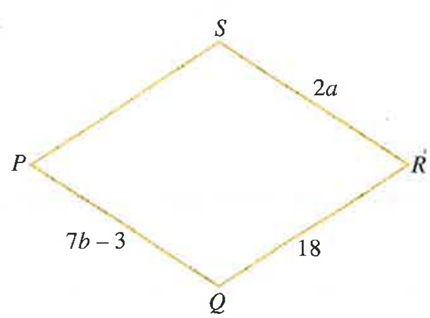
   (b)



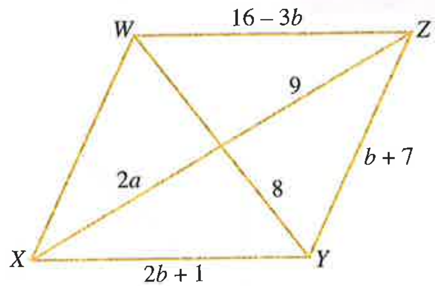
5. Find the values of a and b in each of the following diagrams.
   (a) $ABCD$ is a rectangle.



   (b) $PQRS$ is a rhombus.



(c) $WXYZ$ is a parallelogram.



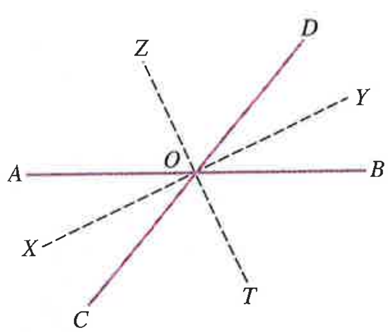
6. It is given that $m\angle A = 50°$, $m\angle B = 50°$, $m\angle C = 80°$, $PQ = 6.5$ cm, $QR = 6$ cm, and $PR = 2.5$ cm.
   - **(a)** Determine which triangle, $\triangle ABC$ or $\triangle PQR$, can be constructed.
   - **(b)** Construct the triangle.
   - **(c)** Classify the triangle by its sides and angles. State the measurement of any relevant sides or angles to support your answer.

7. **(a)** Using a ruler and compasses, construct
   - **(i)** $\triangle ABC$ with $AB = 3$ cm, $BC = 3.5$ cm, and $AC = 5$ cm,
   - **(ii)** the perpendicular bisectors of the three sides of $\triangle ABC$, and labeling the point where the bisectors meet with the letter E.

   (b) What can you say about the lengths AE, BE and CE?

   (c) Draw a circle, with center E, which passes through the vertices of $\triangle ABC$.

8. In the diagram, AOB and COD are straight lines such that $m\angle BOD = 50°$.



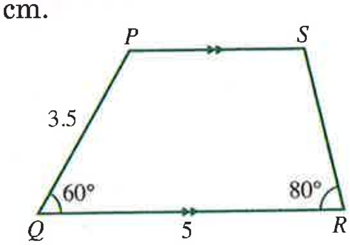
   (a) Using a ruler and compasses, construct
   - **(i)** the line XOY such that the ray OY is the angle bisector of $\angle BOD$,
   - **(ii)** the line TOZ such that the ray OT is the angle bisector of $\angle BOC$.

   (b) Is OX the angle bisector of $\angle AOC$? Why?

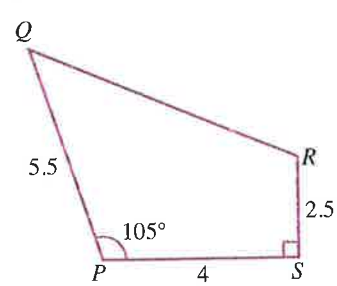
   (c) Find $m\angle XOT$.

9. Construct a rhombus $ABCD$ in which $AC = 5$ cm and $BD = 6$ cm. Measure and write down
   - **(a)** the length of CD,
   - **(b)** the size of one of its acute angle.

10. Construct a trapezoid $PQRS$ in which PS is parallel to QR, $PQ = 3.5$ cm, $QR = 5$ cm, $m\angle PQR = 60°$, and $m\angle QRS = 80°$.
    - **(a)** Construct the perpendicular bisector of PS.
    - **(b)** If the perpendicular bisector of PS meets QR at T, measure TQ and give your answer correct to the nearest 0.1 cm.



11. Construct a quadrilateral $PQRS$ in which $PQ = 5.5$ cm, $PS = 4$ cm, $RS = 2.5$ cm, $m\angle SPQ = 105°$, and $m\angle PSR = 90°$.
    - **(a)** Construct the angle bisector of $\angle PQR$.
    - **(b)** Construct the perpendicular bisector of QR.
    - **(c)** If the bisectors from **(a)** and **(b)** intersect at W, measure WS and give your answer correct to the nearest 0.1 cm.



IN A NUTSHELL

Circle

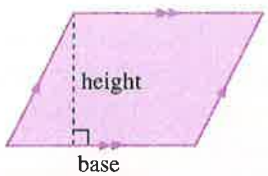
$$\text{Diameter} = 2 \times \text{Radius}$$

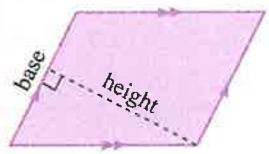
$$\text{Circumference} = \pi \times \text{Diameter}$$

$$= 2 \times \pi \times \text{Radius}$$

$$\text{Area} = \pi \times (\text{Radius})^2$$

Parallelogram

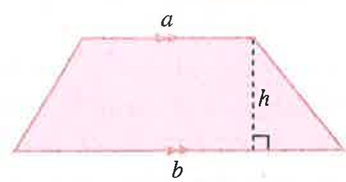




Area = Base × Perpendicular height

The base and height must be perpendicular to each other.

Trapezoid

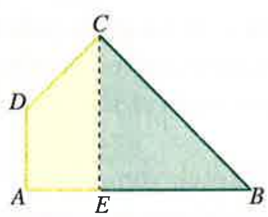


$$\text{Area} = \frac{1}{2} \times \text{Sum of parallel sides} \times \text{Height}$$

$$= \frac{1}{2} \times (a + b) \times h$$

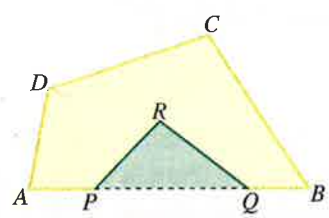
Composite Figures

(a) Using Sum



Area of $ABCD$
= Area of $AECD$ + Area of $\triangle BCE$

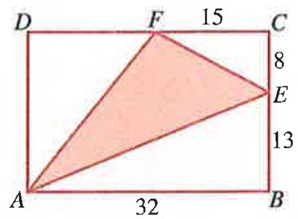
(b) Using Difference



Area of $APRQBCD$
= Area of $ABCD$ – Area of $\triangle PQR$

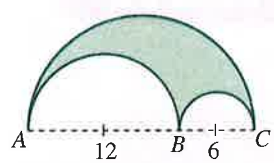
REVIEW EXERCISE 12

1. In the figure, $ABCD$ is a rectangle in which $AB = 32$ cm, $BE = 13$ cm, $EC = 8$ cm, and $CF = 15$ cm. Find
(a) the perimeter of $ABCD$,
(b) the area of $ABCD$,
(c) the area of $\triangle AEF$.

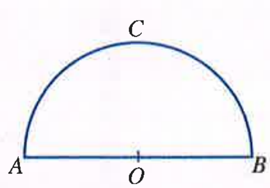


2. A circular pie was cut into five equal pieces. Each piece of pie has an area of 63 cm^2. Find the perimeter of each piece of pie, giving your answer correct to 2 decimal places.

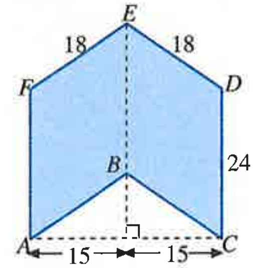
3. In the figure, the shaded region is formed by three semicircles with $AB = 12$ cm, and $BC = 6$ cm. Find, in terms of π,
(a) the perimeter,
(b) the area
of the shaded region.



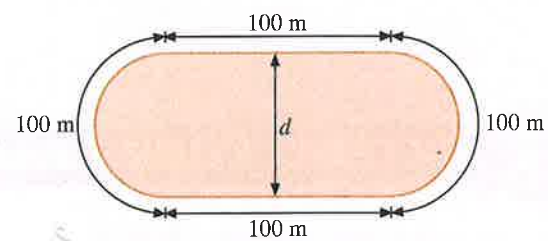
4. The figure shows a semicircle of area 308 cm^2 formed by a piece of wire.
(a) Find the radius of the semicircle.
(b) Find the perimeter of the semicircle.
(c) If the semicircle is cut at A and bent into a circle, find the radius of the circle.
Round your answers to 2 decimal places.



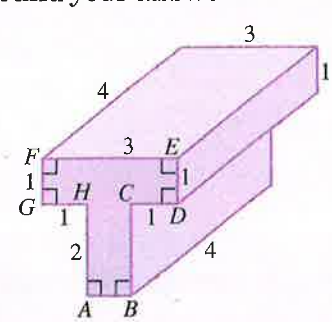
5. In the figure, $ABEF$ and $BCDE$ are two parallelograms. $CD = 24$ cm, $DE = EF = 18$ cm, and $AC = 30$ cm. Find
(a) the perimeter of the figure,
(b) the area of the figure,
(c) the perpendicular distance from F to AB.



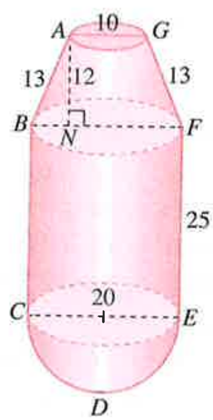
6. The figure shows the innermost line of a 400-meter running track, with two straight sides of length 100 m each and two semicircles of length 100 m each. Find
(a) d, the distance between the two straight sides,
(b) the area enclosed by the track.
Round your answers to 2 decimal places.



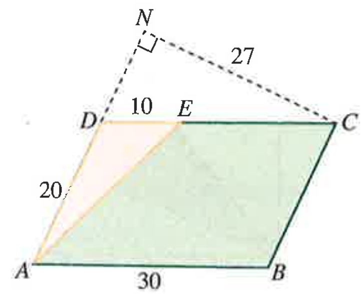
7. The figure shows a solid with a uniform cross-section. $CD = DE = FG = GH = 1$ cm, $AH = 2$ cm, and $EF = 3$ cm.
(a) Sketch and label the cross-section of the solid.
(b) Find the perimeter and area of the cross-section.
(c) If a square has the same area as the cross-section of the given solid, what is the length of a side of the square? Round your answer to 2 decimal places.



8. The figure shows the internal part of a thermal bottle. The uniform cross-section of the bottle consists of a trapezoid, a rectangle, and a semicircle. $AB = GF = 13$ cm, $CE = 20$ cm, $EF = 25$ cm, $AG = 10$ cm, and $AN = 12$ cm.
   (a) Draw and label the cross-section $ABCDEFG$ of the bottle.
   (b) Find the perimeter and area of the cross-section, giving your answer correct to the nearest integer.



9. In the figure, $ABCD$ is a parallelogram in which $AB = 30$ cm, $AD = 20$ cm, $DE = 10$ cm, and $CN = 27$ cm. Find
   (a) the perimeter of $ABCD$,
   (b) the area of $ABCD$,
   (c) the area of $ABCE$,
   (d) the ratio of the area of $\triangle ADE$ to the area of $ABCE$.



EXTEND YOUR LEARNING CURVE

Maximum Area with Given Perimeter

1. You are given a piece of wire of a certain length. Your task is to bend the wire into a rectangle with the greatest possible area. What type of rectangle do you think it should be? Explain your answer.

2. Which shape gives the greatest possible area for a given perimeter?

WRITE IN YOUR JOURNAL

Write what you understand about the concept of area and perimeter. Describe the application of the concept of area to a real-life situation.

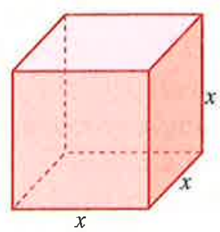
IN A NUTSHELL

Net

A net of a solid is a plane figure that can be folded up to form the solid.

Cube

A cube has six equal square faces.



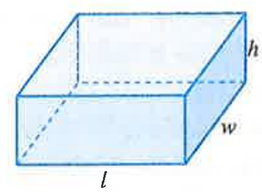
Volume $= x^3$

Total surface area $= 6x^2$

where x is the length of a side of the cube.

Cuboid

A cuboid has six rectangular faces.



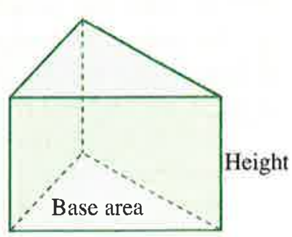
Volume $= lwh$

Total surface area $= 2(lw + lh + wh)$

where l, w, and h are the length, width, and height of the cuboid respectively.

Prism

A prism is a solid with two parallel polygonal bases and a uniform cross-section.



Volume = Base area × Height

Total surface area
= Perimeter of the base × Height
+ 2 × Base area

Unit Conversion

$1 \text{ m}^2 = 10{,}000 \text{ cm}^2$

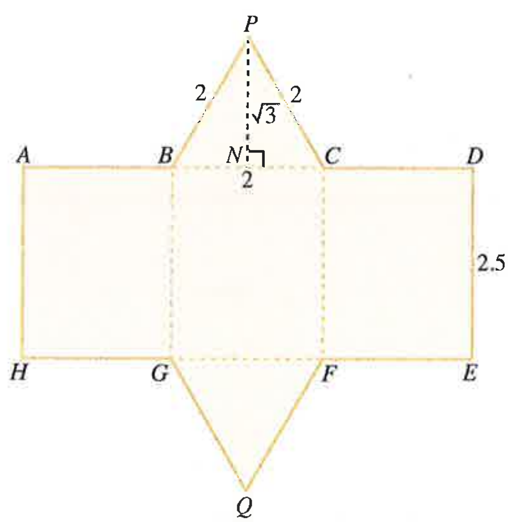
$1 \text{ m}^3 = 1{,}000{,}000 \text{ cm}^3$

REVIEW EXERCISE 13

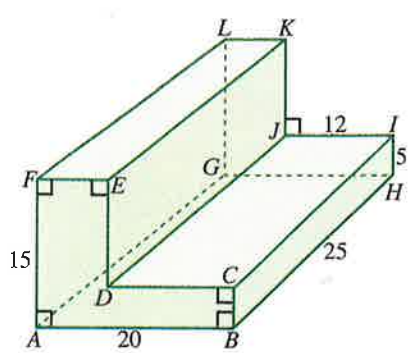
1. A cuboid is 3.5 cm long, 2 cm wide, and 1.5 cm high.
   (a) Draw a net of the cuboid.
   (b) Find the volume of the cuboid.
   (c) Find its total surface area.

2. The face of a cube has an area of 196 in^2. Find
   (a) the length of a side of the cube,
   (b) its total surface area,
   (c) its volume.

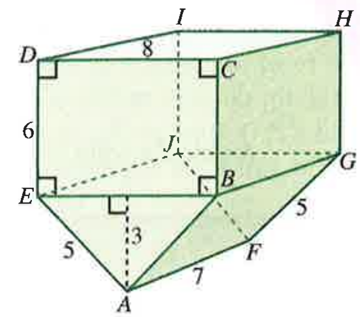
3. The figure shows the net of a prism. $BC = PB = PC = 2$ cm, $DE = 2.5$ cm, and $PN = \sqrt{3}$ cm.
   (a) Name the prism.
   (b) Find the volume of the prism.
   (c) Find its total surface area.



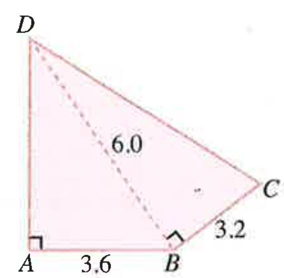
4. Find the volume and surface area of each prism, where the unit of length is cm.
   (a)



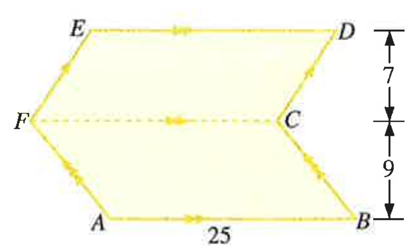
   (b)



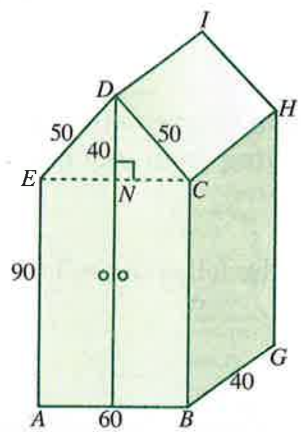
5. The figure shows the cross-section $ABCD$ of a prism of height 5.0 cm. $AB = 3.6$ cm, $BC = 3.2$ cm, $BD = 6.0$ cm, and $m\angle BAD = m\angle CBD = 90°$.
   (a) Construct the quadrilateral $ABCD$ using a protractor, a pair of compasses, and a ruler.
   (b) Measure the lengths of AD and CD.
   (c) Draw a net of the prism.
   (d) Find the volume of the prism.
   (e) Find its total surface area.



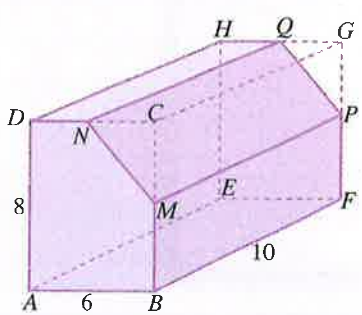
6. The figure shows the cross-section of a solid metal prism of height 20 cm.
   (a) Find the volume of the prism.
   (b) If the prism is melted and recast into a cuboid of length 30 cm and width 16 cm, find the height of the cuboid.
   (c) If the prism is melted and recast into a cube, find the length of a side of the cube.



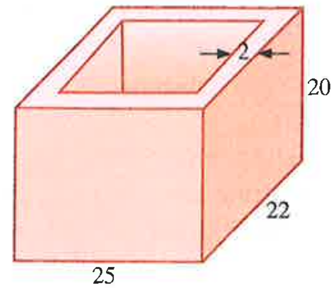
7. The figure shows a cabinet whose uniform cross-section is a triangle CDE on a rectangle $ABCE$. $AB = 60$ cm, $AE = 90$ cm, $DN = 40$ cm, $DC = DE = 50$ cm, and $BG = 40$ cm. Find
   (a) the perimeter of the cross-section,
   (b) the area of the cross-section,
   (c) the surface area of the cabinet, not including the bottom area in contact with the floor,
      (i) in square centimeters,
      (ii) in square meters,
   (d) the volume of the cabinet
      (i) in cubic centimeters,
      (ii) in cubic meters.



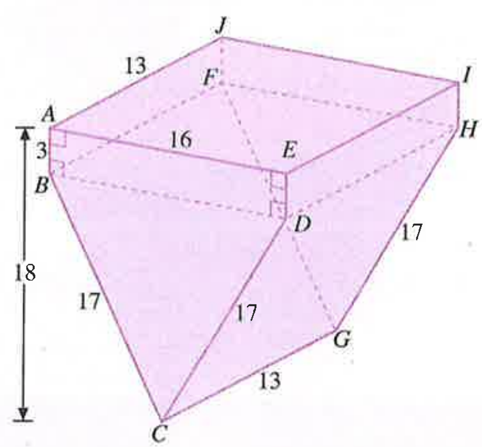
8. In the figure, a triangular prism $CMNGPQ$ is cut off from a cuboid $ABCDEFGH$. $AB = 6$ cm, $AD = 8$ cm, $BF = 10$ cm, M and N are the midpoints of BC and CD respectively.
   (a) Find the volume of the cuboid.
   (b) Find the volume of the triangular prism.
   (c) Find the volume of the remaining prism.
   (d) Find the percentage decrease in the volume of the cuboid due to the removal of the triangular prism.
   (e) Is the total surface area of the remaining prism greater than that of the cuboid? Explain briefly.



9. The figure shows an open wooden box. Its external length, width, and height are 25 cm, 22 cm, and 20 cm respectively. The thickness of the wood is 2 cm. Find
   (a) the volume of wood used in making the box,
   (b) the total surface area of the box.



10. The figure shows an open water container whose uniform cross-section is a rectangle $ABDE$ on a triangle BCD. The length of the container is 13 cm.
   (a) Draw a net of the prism.
   (b) Find the total outer surface area and the volume of the container.
   (c) If water flows at a rate of 14 cm^3/s into the empty water container, how many minutes will it take to fill the container completely?



IN A NUTSHELL

Scale Drawings

Scale drawings are reduced or enlarged drawings of actual objects.

Scale of a Drawing

The scale of a drawing can be expressed as a ratio $1 : n$ where 1 unit length on the drawing represents an actual length of n units.

Map Scale

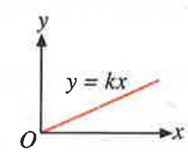
- Scale $=$ Map length : Actual length

  $= 1 : r \left(\text{or } \frac{1}{r}\right)$
- Area on a map : Actual area $= 1 : r^2$

Direct Proportion

When two quantities, x and y, are in direct proportion:

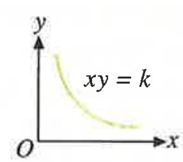
- $y = kx \left(\text{or } \frac{y}{x} = k, x \neq 0\right)$, where k is a constant
- the graph of y against x is a straight line passing through the origin
- $\frac{y_1}{y_2} = \frac{x_1}{x_2}$, where (x_1, y_1) and (x_2, y_2) are any two pairs of values of x and y, $x_2 \neq 0$ and $y_2 \neq 0$

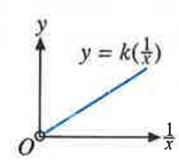


Inverse Proportion

When two quantities, x and y, are in inverse proportion:

- $xy = k$, where k is a constant
- the graph of y against x is part of a curve called a hyperbola
- the graph of y against $\frac{1}{x}$ is part of a straight line passing through the origin but (0, 0) is not inclusive on the graph
- $x_1y_1 = x_2y_2$, where (x_1, y_1) and (x_2, y_2) are any two pairs of values of x and y

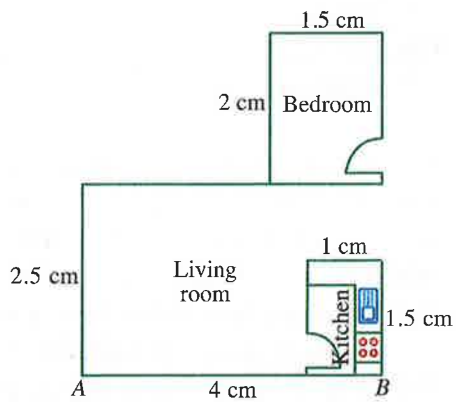




REVIEW EXERCISE 14

1. The shaft of a gardening tool has a length of 25 cm and a diameter of 3.8 cm. It is drawn 5 cm long in a mechanical drawing. Find
   (a) the scale of the drawing,
   (b) the diameter of the shaft in the drawing.

2. The diagram shows a part of the floor plan of an apartment. The actual length of the wall, AB, of the living room is 8 m.



   (a) the scale of the floor plan,
   (b) the actual width of the living room,
   (c) the actual dimensions of the bedroom.

3. The scale of a floor plan is 1 in. to $16\frac{2}{3}$ ft. Find
   (a) the scale of the plan,
   (b) the length of a hallway on the plan if its actual length is 50 ft,
   (c) the actual area of a rectangular garden if its dimensions on the plan are 3 in. by $1\frac{1}{2}$ in.

4. A map is drawn to the scale 1 : 4,000.
   (a) The length of a road on the map is 3.5 cm. Find its actual length in meters.
   (b) The area of a garden is 20,000 m^2. Find the area of the garden on the map in square centimeters.

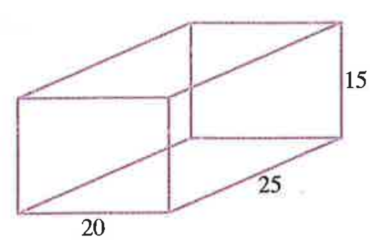
5. The scale of map A is 1 : 5,000 and the scale of map B is 1 : 3,000.
   (a) The distance between two schools on map A is 6 cm. Find their distance apart on map B.
   (b) The area of a lake on map B is 18 cm^2. Find
      (i) the actual area of the lake in m^2,
      (ii) the area of the lake on map A.

6. A carat is a measure of mass used for gemstones. The following table shows the conversion between carats and grams.

Gram (x)	2	4	6	8	10
Carat (y)	10	20	30	40	50

   (a) Show that x and y are in direct proportion.
   (b) Draw the graph of y against x.
   (c) Find the equation connecting x and y.
   (d) The mass of a piece of diamond is 7 g. How many carats is this?

7. The cost $\$C$ of a metal wire is directly proportional to its length x in. When the length is 3 in., its cost is \$0.20.
   (a) Find the length of the wire if its cost is \$15.
   (b) Metal wire is used to make the frame of a rectangular prism that measures 25 in. by 20 in. by 15 in. Find the total cost of the wire used.



8. The mass of a model car is directly proportional to the cube of its length. When the length is 6 cm, its mass is 96 g.
   (a) Find the length of the model car when its mass is 324 g.
   (b) The model car is made using a scale of 1 : 50. The length of the real car is 4 m. Find
      (i) the length of the model car in centimeters,
      (ii) the mass of the model car, correct to the nearest gram.

9. The following table shows some corresponding values of two quantities, x and y.

x	0	1	2	3	4
y	0	2	8	18	32

   (a) Draw the graph of y against x.
   (b) Draw the graph of y against x^2.
   (c) State the relationship between x and y.
   (d) Write down an equation connecting x and y.
   (e) Find the value of y when $x = 2.5$.

10. Suppose the amount of food served to each person at a party is inversely proportional to the number of people who attend the party. When there are 25 people at the party, each person will have 1.2 kg of food.
   (a) If there are 60 people, find the amount of food each person is served.
   (b) How many people are there at the party if each person is served 0.4 kg of food?

11. The dimensions of a rectangle are x in. by y in. It is known that x and y are in inverse proportion. When $x = 36$, $y = 20$.
   (a) Find the equation connecting x and y.
   (b) Find the value of y when $x = 24$.
   (c) Find the value of x when $y = 15$.
   (d) What can you say about the area of the rectangle?

12. The intensity of radiation is inversely proportional to the square of the distance from a radioactive source. When the distance is 4 m, the intensity of radiation is 900 units. Find
   (a) the intensity of radiation when the distance from its source is 5 m,
   (b) the distance from the source if its intensity of radiation is 1,600 units.

13. 6 men take 6 hours to complete a job. The time required to complete the job, T hours, is inversely proportional to the number of workers, x.
   (a) Find the time taken to complete the job when there are 9 workers.
   (b) How many workers are required if the job has to be completed in 45 minutes?

14. In kick boxing, it is found that the force needed to break a board is inversely proportional to the length of the board. If it takes 6 lb of pressure to break a board 2 ft long, how many pounds of pressure will it take to break a board that is 5 ft long?